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MODELING ASBESTOS POPULATIONS: A FRACTAL APPROACH

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ABSTRACT

Distributions of length and mass are important to mineral producers whose products contain trace asbestos and to biological scientists who experiment with asbestos. Analysis of 56 distributions of the length of asbestos fibers shows that length frequency follows a power law, from which the population's fractal dimension can be determined. From empirical observations of width, thickness, and density in combination with length, the frequency of incremental mass can be calculated. For many asbestos samples, the proportion of total mass of an asbestos population increases as the mass and length of individual fibers and bundles of fibers increase. Measurement strategies should be designed to include the longest fibers (SEM or OM) for weight-based abundances. Where a population's mass is concentrated in the shortest fibers, the TEM is the most appropriate instrument for gathering dimensional data. In either case, the application of the fractal model enables the entire mass of the population to be estimated from a random sample, provided the mass of the largest and smallest particles in the population are known or can be estimated. Where asbestos is a contaminant, its abundance can be estimated if the total mass of the sample examined is known.

Keywords: asbestos, asbestos concentration, fractal.

SOMMAIRE

Une connaissance de la distribution de la longueur et de la masse des particules s'avère importante pour les producteurs de minerai dont les produits contiennent des traces d'amiante, ainsi que pour les biologistes qui effectuent des expériences avec de l'amiante. Une analyse de cinquante-six distributions de la longueur de fibres d'amiante démontre que la fréquence des longueurs répond à une fonction à puissance, de laquelle il est possible de déterminer la dimension fractale de la population. A partir d'observations empiriques portant sur la largeur, l'épaisseur et la densité, combinées aux mesures de longueur, il est possible de calculer la fréquence de la masse incrémentielle. Dans le cas de plusieurs échantillons d'amiante, la proportion de la masse totale d'une population augmente à mesure qu'augmentent la masse et la longueur des fibres individuelles et des essaims de fibres. Les protocoles de mesurage devraient inclure les fibres les plus longues (telles que mesurées au microscope électronique à balayage et au microscope optique) pour une caractérisation pondérale d'une population. Dans le cas où la masse d'une population est concentrée dans les fibres les plus courtes, c'est par microscopie électronique par transmission qu'il faudrait caractériser les dimensions de la population. Dans l'un ou l'autre des cas, l'application d'un modèle fractal permet d'estimer les propriétés d'une masse entière à partir d'un échantillon quelconque, pourvu qu'on puisse connaître ou estimer la masse de la particule la plus grande et celle de la plus petite d'une population. Dans les situation où l'amiante agit comme contaminant, son abondance peut être estimée si on connaît la masse totale de l'échantillon.

(Traduit par la Rédaction)

Mots-clés: amiante, concentration d'amiante, fractal.

INTRODUCTION

Asbestos is a term applied to a group of minerals that share a common habit, which is characterized by fibers of several tenths of a micrometer or less in width, referred to as fibrils, that occur in parallel bundles (Steel & Wylie 1981). In the case of both chrysotile and amphibole-asbestos, the fibrils are randomly oriented perpendicular to the fiber axis, and they readily separate by hand pressure. In addition to enhanced tensile strength and flexibility, the asbestiform habit results in certain anomalous optical properties, such as parallel extinction, which are most apparent in the clinoamphiboles (Wylie 1979). For many years, "crocidolite" (riebeckite-asbestos) and "amosite" (grunerite-asbestos) were considered to be orthorhombic because they exhibit parallel extinction. It may also be the case that some amphibole-asbestos is characterized by a high incidence of Wadsley defects (Chisholm 1973, Veblen *et al.* 1977) and stacking disorder (Hutchison *et al.* 1975), but these features do not appear to be essential characteristics of the asbestiform habit (Dorling & Zussman 1987).

All types of commercial asbestos are known to be pathogenic, and in the United States, all are regulated as carcinogens. Building materials are defined by U.S.

Environmental Protection Agency as asbestos-containing if they contain more than 1 wt.% asbestos (U.S. Environmental Protection Agency, 1982), and any commercial material must be labeled if it contains more than 0.1 wt.% of a known carcinogen (U.S. Occupational Safety and Health Administration, 1983). Because it is habit, not crystal structure or chemical composition, that defines asbestos, bulk analytical techniques such as X-ray diffraction, infrared absorption spectroscopy, and bulk chemical analysis usually cannot be used to determine concentration (unless it is known in advance that the asbestiform variety is the only form of the mineral present). Therefore, an analyst must usually rely on microscopy. An application of microscopy, whether it is optical (OM), scanning electron (SEM) or transmission electron microscopy (TEM), to the determination of concentration, greatly benefits from knowledge of the nature of the size distribution of the component, *e.g.*, normal, log normal, fractal, *etc.*, and the range over which the size of the fibers varies.

Researchers who study carcinogenicity of mineral fibers are also concerned with the size and shape of asbestos fibers. Stanton *et al.* (1981) and others have shown that size and shape are important variables in predicting carcinogenic potential of inorganic materials implanted in animals. The dose of mineral fiber is commonly reported as the number of fibers of a particular range of length and width per milligram of implanted sample. Establishment of such a dose is highly dependent upon accurate assessment of the size distribution of the particles in the sample. Populations of fibers represent special problems in this regard because of their anisotropic dimensions.

For almost any population of asbestos fibers, lengths range over several orders of magnitude, and the shorter fibers are always much more abundant than the longer ones. For most populations, the majority of the mass is tied up in the largest, least abundant particles, but for some, the opposite is the case. For weight-based abundances, it is essential that the population characteristics be predicted accurately from the sample characteristics, and to do so, it is necessary to know how length, width, and thickness are distributed in the population.

To overcome the problems of characterizing populations of mineral fibers, models fitting the distributions of length, width, and thickness have been constructed. Log width has been shown to be a linear function of log length for populations of asbestos and cleavage fragments of some elongate minerals (Siegrist & Wylie 1980), and log thickness is a linear function of log width for riebeckite-asbestos and grunerite-asbestos (Wylie *et al.* 1982). It has also been shown that whereas distributions of log width and log length may resemble log normal distributions superficially, in most cases, this model is not statistically valid (Siegrist & Wylie 1980).

Whereas some of the relationships such as those mentioned above can be very useful in modeling asbestos populations, the distributions of length and mass have not been shown to follow any consistent model. Consequently, it has been general practice in the characterization of asbestos samples to measure length and width of some number of randomly selected particles, usually between 200 and 1000, to calculate the mass of each by assuming some density and cross-sectional shape, usually a circle with diameter equal to width, and then to sum the masses of all measured particles. By assuming that the random selection is indicative of the whole, the number of particles per unit mass in specified dimensional categories, as well as the total mass of asbestos, are estimated from the sample measurements.

Such a measurement strategy probably produces a distribution of length that is representative of the population on the basis of particle number, but it may not provide a very accurate assessment of the distribution of mass within the population. Because of the abundance of short particles, only a few of the longest particles, in which a significant amount of mass may be concentrated, are usually included in the measured sample. In other populations, the proportion of the mass of the population may increase as length decreases because of rapidly increasing numbers of fibers, but the analytical techniques or sampling protocol (or both) may preclude inclusion of the smallest fibers. Without knowing whether the largest or shortest fibers contain the bulk of the mass, the standard strategy of measurement can lead to significant error in estimating the abundance of asbestos on a weight basis. Furthermore, without a model to which the distribution of dimensional data from a sample can be fit, there is no way to assess the representativeness of the sample or to extrapolate to unmeasured portions of the population.

In this paper, I will describe a model for the distribution of the length of asbestos fibers, the dimension most readily measured. Cross sections and density can be obtained by measurement or approximation. The total mass of a given aggregate of fibers is then integrated over the range of length found in the aggregate, and the proportion of mass contained in individual ranges of length or mass can be estimated.

The Fractal Approach

There are a variety of scale-invariant processes in nature. In particular, fragmentation has been clearly demonstrated to follow a power law. The concept of fractals as proposed by Mandelbrot (1967) provides a means of quantifying these processes (Turcotte 1986, Feder 1988). A self-similar fractal is defined by the relationship:

$$N = Cr^{-D} \quad (1)$$

where N is the number of objects with a particular dimension greater than r, C is a constant, and D is the fractal dimension. This relationship can be expressed also as:

$$\log N = -D \log r + C \tag{2}$$

Turcotte (1986) summarized the studies that have been done on the fractal distribution of natural materials caused by fragmentation. The populations that he reported range from the fragmentation of gabbro by a lead projectile, for which D is equal to 1.44 (Lange *et al.* 1984), to broken coal (D = 2.50) (Bennett 1936), to ash and pumice (D = 3.54) (Hartmann 1969). Turcotte concluded that the fractal dimension is a measure of the resistance of the material to fragmentation, such that the lower the fractal dimension, the less is the resistance offered by the material.

The fibrillar habit of asbestos results in fiber bundles that are easily disaggregated. However, disaggregation does not involve breaking the strong bonds that form the mineral structure, although it may involve breaking weak hydrogen or Van der Waals bonds that form between fibrils. Furthermore, asbestos fibrils possess extreme tensile strength parallel to the fiber axis and are difficult to break perpendicular to this direction. Because of the extreme anisotropy in strength, therefore, analogies between disaggregation and fracturing of a rock may not be appropriate. Nonetheless, as this paper will show, Turcotte's conclusions have some applicability to asbestos since the relatively low fractal dimensions characteristic of all types of asbestos are consistent with the property of disaggregation under hand pressure.

In order to test the appropriateness of the fractal model for asbestos fiber populations, 56 published distributions of the length of various samples of asbestos were used to test the model (see Table 1 for references). Represented among these populations are samples of airborne and bulk asbestos, and asbestos fibers from lung tissue. Dimensional data were collected by TEM, SEM, and OM at magnifications that range from about 5,000 to 100,000 × for TEM, 1,000–20,000 for SEM, and 120–600 for OM. The minerals represented are riebeckite, grunerite, chrysotile, anthophyllite and tremolite. The data are published as frequency of fibers in particular categories of length that reflect the range of lengths measured. The samples vary from a few hundred to a few thousand particles.

The fractal dimension D of a population is taken as the slope of the least-squares linear regression line where the dependent variable is N, the log number of particles with length greater than r, and the independent variable is log length. The regression was applied to the full range of length data, with the following two exceptions. First, length data gathered primarily from OM and SEM measurements commonly show a smaller number of fibers in the smallest length categories (length less than a few μm) than in the immediately adjacent categories of length. This dropoff in the number of small fibers represents, at least in part, limitations in instrumentation and cannot be considered to be representative of the population as a whole. Therefore, in 22 of the 56 samples, two or more of the smallest-length categories were excluded from the regression analysis. Second, in

TABLE 1. THE FRACTAL DIMENSIONS OF THE LENGTH OF ASBESTOS FIBERS

Sample	Instr.	Range of log length (μm)	R^2	D
Riebeckite-asbestos				
a. NIEHS bulk[a]	OM	(-0.05) 0.74 - 2.50	0.98	1.43
b. NIEHS bulk[a]	SEM	(-1.00) 0.70 - 2.00	0.99	1.34
c. Cape S.A. air[c]	TEM	(0.00) 0.00 - 1.00	0.94	1.71
d. Cape S.A. lung[c]	TEM	(-0.70) 0.00 - 1.00	0.94	1.47
e. UICC bulk[d]	OM	(0.30) 0.30 - 1.60	0.99	1.88
f. UICC bulk[d]	OM	(0.30) 0.30 - 1.90	>0.99	1.74
g. UICC air[d]	TEM	(-0.70) 0.00 - 1.30	0.98	1.32
h. UICC air[d]	OM	(0.30) 0.60 - 1.78	0.95	1.50
i. Australia 1 bulk[e]	TEM	(-1.30) -1.30 - 1.00	0.94	0.69
j. Australia 1 bulk[e]	TEM	(0.30) 0.30 - 1.70	0.99	1.62
k. Australia 2 bulk[e]	TEM	(-1.10) -1.10 - 1.00	0.99	0.52
l. Australia 2 bulk[e]	TEM	(0.30) 0.30 - 1.70	0.98	0.98
m. Bolivia 1 bulk[e]	TEM	(-1.05) -1.05 - 1.00	0.95	0.37
n. Bolivia 1 bulk[e]	TEM	(0.30) 0.30 - 1.70	0.96	0.58
o. Cape S.A. 1 bulk[e]	TEM	(-1.10) -1.10 - 1.00	0.94	1.09
p. Cape S.A. 1 bulk[e]	TEM	(0.30) 0.30 - 1.48	0.96	2.08
q. Cape S.A. 2 bulk[e]	TEM	(-0.85) -0.85 - 1.00	0.95	1.49
r. Cape S.A. 2 bulk[e]	TEM	(0.30) 0.30 - 1.70	0.95	1.70
s. Transvaal 1 bulk[e]	TEM	(-0.92) -0.92 - 1.00	0.96	0.45
t. Transvaal 1 bulk[e]	TEM	(0.30) 0.30 - 1.70	>0.99	0.58
u. Transvaal 2 bulk[e]	TEM	(-1.00) -1.00 - 1.00	0.96	0.70
v. Transvaal 2 bulk[e]	TEM	(0.30) 0.30 - 1.70	0.99	1.09
w. NIEHS Tr. 3 bulk[e]	TEM	(-1.05) 0.28 - 1.00	0.98	1.55
x. NIEHS Tr. 3 bulk[e]	TEM	(0.30) 0.30 - 1.70	0.99	1.65
y. Transvaal 4 bulk[e]	TEM	(-1.00) -1.00 - 1.00	0.98	0.57
z. Transvaal 4 bulk[e]	TEM	(0.30) 0.30 - 1.70	0.99	0.62
aa. Transvaal 5 bulk[e]	TEM	(-1.15) -1.15 - 1.00	0.95	0.57
bb. Transvaal 5 bulk[e]	TEM	(0.30) 0.30 - 1.70	>0.99	0.76
Chrysotile				
a. NIEHS short bulk[a]	OM	(0.30) 1.00 - 1.15	0.96	3.34
b. NIEHS short bulk[a]	TEM	(0.00) 0.30 - 1.00	0.99	1.79
c. NIEHS long bulk[a]	OM	(-0.05) 0.74 - 3.00	0.93	1.15
d. NIEHS long bulk[a]	SEM	(0.30) 0.30 - 2.78	0.92	0.83
e. Canada lung[c]	TEM	(0.00) 0.00 - 1.00	>0.99	1.35
f. R. UICC bulk[d]	OM	(0.30) 0.30 - 1.90	>0.99	1.57
g. R. UICC bulk[d]	OM	(0.30) 0.30 - 1.78	0.97	1.50
h. R. UICC air[d]	OM	(0.30) 0.60 - 1.78	0.99	1.22
i. R. UICC air[d]	TEM	(-0.70) 0.30 - 1.30	0.97	1.56
j. C. UICC bulk[d]	OM	(0.30) 0.30 - 1.90	0.99	1.31
k. C. UICC bulk[d]	OM	(0.30) 0.30 - 1.78	0.92	1.46
l. C. UICC air[d]	OM	(0.30) 0.60 - 1.70	0.97	1.86
m. C. UICC air[d]	TEM	(-0.70) 0.30 - 1.30	0.97	1.27
Grunerite-asbestos				
a. NIEHS bulk[a]	OM	(0.28) 0.59 - 2.30	0.93	1.12
b. NIEHS bulk[a]	SEM	(0.30) 0.78 - 2.78	0.96	1.03
c. Africa lung[c]	TEM	(-0.70) 0.30 - 1.00	0.98	1.33
d. UICC bulk[d]	OM	(0.30) 0.30 - 1.90	0.99	1.60
e. UICC bulk[d]	OM	(0.30) 0.30 - 1.78	0.98	1.89
f. UICC air[d]	OM	(0.30) 0.30 - 1.78	0.99	1.92
g. UICC air[d]	TEM	(-0.70) 0.30 - 1.30	0.96	1.05
Tremolite-asbestos				
a. Calif. >3:1 bulk[f]	SEM	(-1.00) 0.30 - 1.00	0.99	1.70
b. Calif. <3:1 bulk[f]	SEM	(-1.00) -0.52 - 0.48	0.99	2.13
c. Korea >3:1 bulk[f]	SEM	(-1.00) 0.00 - 1.00	0.98	1.42
d. Korea <3:1 bulk[f]	SEM	(-1.00) -0.52 - 0.48	0.99	1.42
Anthophyllite-asbestos				
a. UICC bulk	OM	(0.30) 0.30 - 1.90	0.99	1.53
b. UICC bulk	OM	(0.30) 0.30 - 1.90	0.99	1.43
c. UICC air	OM	(0.30) 0.45 - 2.04	>0.99	1.96
d. UICC air	TEM	(-0.70) 0.50 - 1.80	0.95	1.19

[a] Campbell et al. (1980). [b] Number in parentheses gives the log of the smallest length in the data set. Numbers outside the parentheses represent the range of lengths over which D was derived. [c] Pooley & Clark (1980). [d] Timbrell (1970). [e] Shedd (1985). [f] Davis et al. (1990).

most of the 56 samples, the smallest-length category had to be excluded from the regression analysis because the investigators do not specify the length of the smallest fiber measured, reporting instead the number of fibers with length less than some value r, where r is typically 1 to 0.1 μm. Because the regression was framed on the number with length greater than r, this datum remained undefinable. The range over which the regression was applied is given in Table 1. The log of the minimum length given in the raw data is given in parentheses.

In all 56 cases, a power law model for the distribution of length was determined and found to be highly significant (90% confidence interval), with a correlation coefficient (R^2) greater than 0.90. Forty-eight populations have an R^2 equal to or greater than 0.95, and in 31 cases, it is equal to or greater than 0.98. R^2 values are given in Table 1. These data indicate that the relationship between number of fibers and fiber length is scale-invariant, and equation 1 holds for asbestos over certain ranges of length.

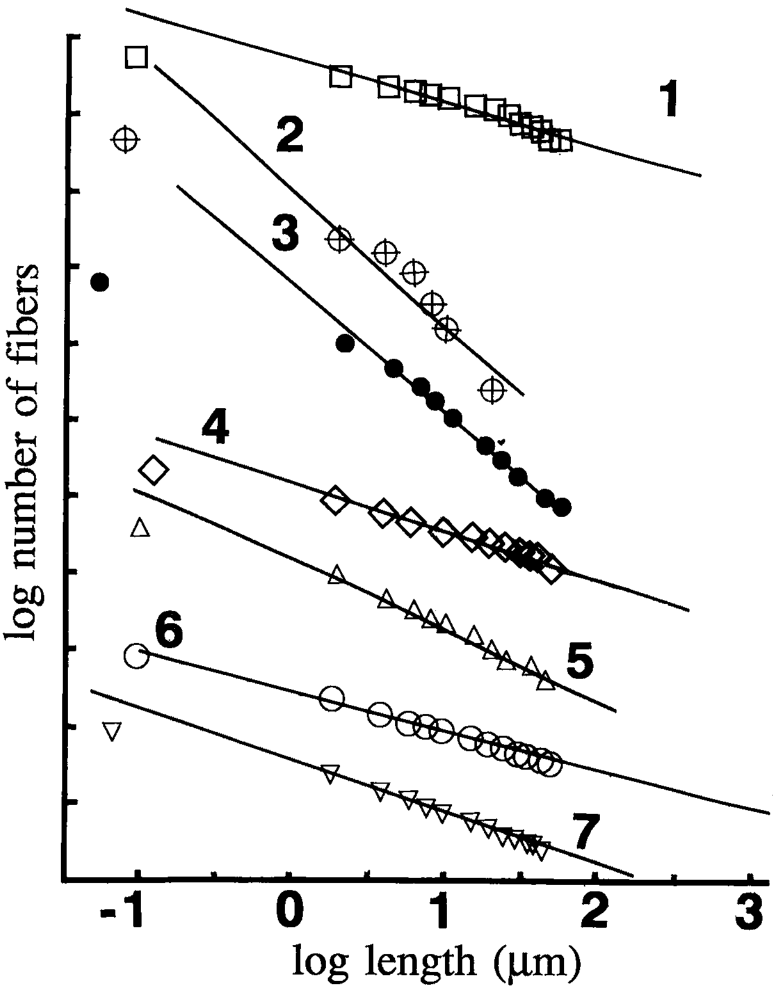

FIG. 1. Log number of fibers with log length greater than indicated magnitude for seven samples of riebeckite-asbestos. Data are derived from Shedd (1985) by combining frequencies for all lengths and for lengths greater than 2 μm assuming that there were equivalent numbers of fibers with lengths between 2 and 10 μm. Curve 1 pertains to samples from Bolivia, curve 2, to samples from the Cape Province, South Africa, curve 3, to samples from the Hamersley Range, Western Australia, and curves 4–7, to samples from the Transvaal, South Africa. Divisions of the vertical axis are whole numbers of log units, with the maximum number of fibers in all populations between log N = 3 and log N = 4.

Whether length distributions are fractal over the entire range of length found in asbestos samples is not entirely clear from the data. Figure 1 illustrates the correspondence of the fractal model with the cumulative frequency distribution for seven riebeckite-asbestos populations reported by Shedd (1985). For the samples shown in this figure and for all the populations reported in Table 1, the model appears to fit well for all lengths greater than a few micrometers. However, it is also clear from Figure 1 that the number of particles in the shortest length category (less than about 1 μm) is generally less than would be predicted from the power law distribution suggested by a fractal model. The differences between observed and expected numbers of fibers are highly variable among samples and may represent limitations in the measurements.

Deviation from the fractal model in the shortest categories of length may arise for three reasons: (1) fibers with length/width less than n may be excluded, (2) portions of the populations may not be visible, and (3) there may be a real decrease in the number of short fibers. Fibers with a length-to-width ratio less than some number, usually 3 or 5, are normally excluded in the sampling protocol. Such aspect-ratio limitations arise from federal regulations that define fibers based on aspect ratio. This means that some proportion of the shortest fibers will not be counted because their aspect ratio is simply not large enough. Davis *et al.* (1990) provided length data for particles both greater than and less than 3:1 in aspect ratio for two samples of tremolite-asbestos. The fractal dimensions are very close for the two groups of aspect ratio, even though the ranges in length overlap very little. These data suggest that, to a large degree, the particles having a greater than and less than 3:1 aspect ratio are actually part of the same population; to combine them would produce a sample that fits the model over the entire range of aspect ratio.

Fiber "visibility" is an important factor in skewing the data toward the longer lengths. It is clearly significant in OM and SEM measurements (which will be discussed later), but it may also play a role if TEM is employed. At 10,000, the image of fibers less than 0.1 μm in length will be less than a millimeter in size and may simply be overlooked or ignored. Whether these shortest particles will be counted also may be a function of their width. In Figure 1, samples 2 and 3 show the greatest uncertainty relative to regression of log number *versus* log length. The two riebeckite-asbestos samples are from the Cape Province of South Africa and the Hamersley range in Australia, respectively. Riebeckite-asbestos from both localities has a narrower width than riebeckite-asbestos from Bolivia (curve 1), or the Transvaal region of South Africa (curves 4–7). The narrower widths (and associated thicknesses) may limit the visibility of the shortest fibers. It seems reasonable to conclude that the Transvaal samples approach an ideal fractal distribution because the smallest fibers were wide enough to be detected.

It can be assumed that in all populations of fibers, there is a lower limit of length. At the extreme is a single unit-cell of about 10^{-3} μm, below which a mineral cannot exist. Probably a few hundred unit-cells are necessary for a mineral to develop properties recognizable on the TEM (10^{-1} μm), so that between these two lengths, the number of fibers must decrease. For all populations, the fractal dimension will approach 0 as the number of fibers in the shortest length categories approaches zero. It may also be that some asbestos populations are multifractal, owing to a change in the fractal dimension at some length. Such behavior might reflect samples composed of more than one mineral or mineral habit. In Figure 1, the behavior of crocidolite 2 may be multifractal, since there appears to be an abrupt change in slope at log length = 0.8 μm. However, most of the asbestos populations I have examined do not exhibit this characteristic.

FRACTAL DISTRIBUTIONS AND MASS FRACTIONS

Instrumentation

Logarithmic plots of mineral populations consistent with the fractal model are useful to compare the three instruments typically used to gather dimensional data: TEM, SEM, and OM. Figure 2 shows the fractal model and the actual data on cumulative distribution for a sample of riebeckite-asbestos (referred to as the National Institute of Environmental Health Science (NIEHS) riebeckite-asbestos (Campbell *et al.* 1980) collected by OM, SEM, and TEM. The fractal dimensions of the three populations are similar: 1.43(OM), 1.33(SEM), and 1.58(TEM). In the samples studied in this paper, the fractal dimensions of samples from the same locality derived from data obtained with different instrumentation are within ± 0.4 of the mean value. Other factors being equal, the most precise estimate of the fractal dimension is probably that derived from data with the widest range in length.

Figure 2 shows that the data begin to deviate significantly from the power law model for lengths less than about 10 μm for both the SEM and OM data. For the TEM, however, the data obey the power law distribution for all lengths greater than about 2 μm. Because the distributions of fibers between 2 and 10 μm are fractal if studied by TEM, it is reasonable to assume that the deviation in this range in the SEM and OM data is due to detection limitations or sampling protocol rather than real deviations in the sample.

Siegrist & Wylie (1980) gave the relationship between log width and log length for the NIEHS riebeckite-asbestos as:

$$\log \text{width} = 0.142 \log \text{length} - 0.709. \tag{3}$$

Equation 3 predicts average widths of 0.3 μm for

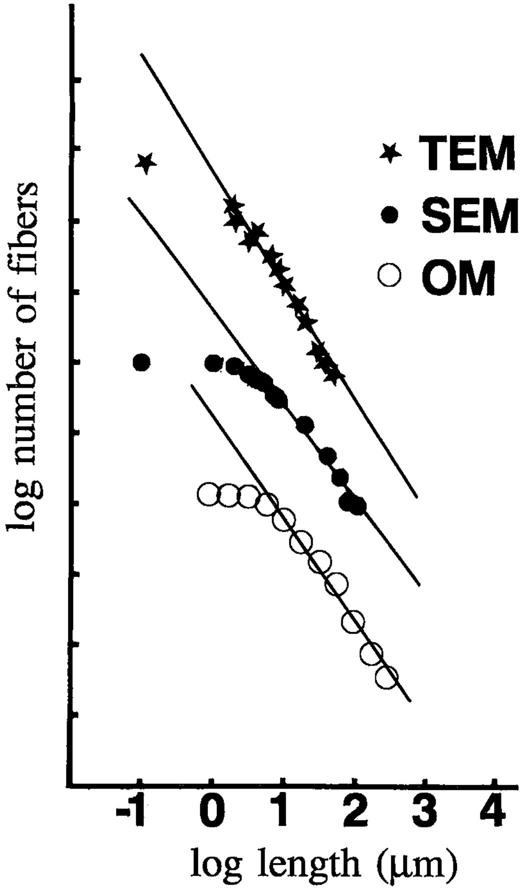

FIG. 2. Log number of fibers with log length greater than indicated magnitude for a sample of riebeckite-asbestos measured by using TEM, SEM and OM. Data from Table 1, samples w, a, and b. Divisions of the vertical axis are whole numbers of log units. The maximum number of fibers in all populations is between log N = 3 and log N = 4.

fibers 10 μm in length. A value of 0.3 μm approximates the limit of resolution of the optical microscope under most conditions, and, for this reason, I would expect OM data to exclude many fibers less than 10 μm in length.

With the SEM, however, image resolution is not a problem. Contrast with the background appears to be the important uncertainty. Figure 2 shows that short fibers are unlikely to be seen, even though widths of 0.3 μm exceed the theoretical resolution of most modern instruments.

The data shown in Figure 2 reinforce what is generally known about the comparability of SEM and OM data: they are similar, because small fibers cannot be resolved by the optical microscope and do not provide sufficient contrast to be detected in commonly employed SEM techniques. It may be possible to improve the visibility of fibers in the SEM, but for the data presented in Table 1, the SEM provides accurate data on population characteristics over about the same range of lengths and widths as the OM. Because of greater resolution and the improved contrast (and visibility) obtained with the TEM, accurate characteristics of the population for the smaller lengths are more likely with the TEM. However, it is also evident in Figure 2 that TEM data omit the longest fibers in the population. For all samples in Table 1 examined by OM or SEM and TEM, there were longer fibers in the OM or SEM data.

The distribution of mass

If we assume that the distributions of number *versus* length are fractal, it is possible to use this model in estimating the distribution of mass by combining it with other models that have been established for the relationships between width and length and between width and thickness. From the relationships among width, thickness and length, it is possible to predict volume as a function of length and, in combination with density, mass as a function of length. From the relationship between mass and length and number and length, number *versus* incremental mass can be established. If an upper and lower limit in the mass of fibers and fiber bundles in a population are known (or can be estimated), the partitioning of the mass in the population can be predicted. In some populations of asbestos, fiber bundles of greatest mass are so rare and small fibers so abundant that the proportion of mass increases as fiber mass (and length) decrease. In other populations, the opposite is observed. How mass is proportioned is an important consideration in formulating an approach to the measurement of abundances in weight percent. In particular, it can be the most important characteristic of a population to determine whether one or all of the methods OM, SEM, or TEM are appropriate for the dimensional data needed.

The relationship between width (w) and length (L) in populations of mineral fibers can be expressed by a linear equation (Siegrist & Wylie 1980) of the form:

$$\log w = \alpha \log L + C_2. \quad (4)$$

Both α and C_2 are derived from a population by least-squares linear regression; α is the regression coefficient, and C_2 is a constant. Similarly, Wylie *et al.* (1982) have shown that for NIEHS grunerite-asbestos and riebeckite-asbestos, the relationship between thickness (t) and width can be expressed by a similar equation:

$$\log t = \beta \log w + C_3 \quad (5)$$

where β is the regression coefficient, and C_3 is a constant. (Note: width is generally greater than thickness because fibers settle out of suspension with their mini-

mum dimension perpendicular to the substrate.) Since mass (m) = density (ρ) x L x w x t, combining Equations 4 and 5 gives log m as a function of fiber length as:

$$\log m = (1 + \alpha + \beta\alpha)\log L + C_4$$

where $C_4 = C_2 + C_3 + \beta C_2 + \log\rho$, or

$$\log m = -B\ (\log L) + C_4 \quad (6)$$

where $B = 1 + \alpha + \beta\alpha$. If fibers were cubes, B would reduce to 3. It is also possible to express the relationship between number of fibers and their incremental mass by combining Equation 6 and Equation 2 so that log N, where N is the number of fibers with mass greater than m, is given as:

$$\log N = -D/B(\log m) - C_4D/B + C.$$

or

$$\log N = -D/B(\log m - C_4) + C. \quad (7)$$

In this case, another fractal relationship is defined by $D_m = D/B$, and Equation 7 can be rewritten in the form:

$$\log N = -D_m\ (\log m - C_4) + C \quad (8)$$

or

$$\log N = -D_m\ (\log m) + D_mC_4 + C \quad (9)$$

D_m, C_4, and C can be calculated for NIEHS riebeckite-asbestos and NIEHS grunerite-asbestos by using the fractal dimensions given in Table 1 and published data on α and β. For some samples of chrysotile and tremolite-asbestos, α is known (Wylie & Schweitzer 1982), but in order to calculate D_m, some relationship between width and thickness must be assumed. In all calculations that follow, with the exception of the NIEHS riebeckite-asbestos and grunerite-asbestos, thickness is assumed to be equal to 0.5 times width. This closely approximates the relationship between width and thickness for NIEHS grunerite-asbestos and riebeckite-asbestos. Table 2 gives the magnitudes of D_m for several populations of asbestos.

It is evident that D_m can be both greater and less than 1 in asbestos populations. Populations of asbestos that are characterized by small fibrils of similar width, such as short-fiber chrysotile and riebeckite-asbestos, have D_m greater than 1. Other types of asbestos that show a much greater variability in width are characterized by D_m less than 1. Grunerite-asbestos, tremolite-asbestos and long-fiber chrysotile (and probably anthophyllite-asbestos) fall in this category; fibril width is variable, and fiber bundles appear to be more tightly bound.

Values of D_m given in Table 2 can be compared to D_m determined from the measured frequency of fiber and

TABLE 2. FRACTAL DIMENSIONS FOR THE DISTRIBUTION OF LENGTH D AND MASS D_m

Sample	D	α	ß	D_m
NIEHS riebeckite-asbestos[a]	1.580	0.142	0.729	1.269
NIEHS grunerite-asbestos[a]	1.033	0.184	0.607	0.797
NIEHS short chrysotile[b]	1.790	0.142	1.000*	1.394
NIEHS long chrysotile[b]	0.833	0.015	1.000	0.809
Korean tremolite-asbestos[c]	1.415	0.246**	1.000	0.943

* Assumed: log thickness = log width - log 2. ** Average of three asbestos samples from the tremolite-actinolite series (Wylie & Schweitzer 1982). [a] Data from Siegrist & Wylie (1980), Wylie et al. (1982), and Campbell et al. (1980). [b] Data from Siegrist & Wylie (1980) and Wylie et al. (1982). [c] Data from Wylie & Schweitzer (1982) and Davis et al. (1990).

fiber bundles. In my laboratory, I have recently completed a study of the distribution of small amounts of tremolite-asbestos in talc ore. Seven samples were studied, and the length and width of fiber bundles of tremolite-asbestos greater than 1 μm in width and longer than 5 μm were recorded. Over one hundred different preparations were examined by three different analysts to obtain the data. (The details of this study are being prepared for publication.) Mass distributions were obtained from measured values of length and width, and from a thickness either measured directly by rolling the fiber bundles or, if this could not be accomplished, from a thickness assumed to be equal to one half the width. From these data, D_m was calculated to be 0.927. This compares remarkably well with $D_m = 0.943$ for the Korean tremolite-asbestos reported in Table 2.

The data given in Table 2 can be used to determine the incremental distribution of mass or weight on a percent basis. To apply these data for this purpose, an upper and lower limit on fiber mass must be established. The upper limit can be taken as the mass of the largest particle in a sample. In practice, this may mean scanning a sample at low magnification to determine the largest bundle of fibers present or predicting the largest bundle of fibers from the largest particle of any type in the sample. For the smallest mass, two approaches can be taken, First, if only those fibers longer than 5 μm are to be considered (federal regulations apply only to asbestos fibers longer than 5 μm), then the mass of a fiber 5 × 0.1 0.05 μm can be calculated for the lower limit of mass. For asbestos, this is approximately 10^{-14} g. (The width and thickness were chosen to approximate the smallest fibers of tremolite-, actinolite- and grunerite-asbestos with a length of 5 μm. In the case of riebeckite-asbestos or chrysotile, dimensions of width and thickness somewhat smaller than this may be used.) If asbestos of any length is to be included in the calculation of weight percent, the smallest mass of any asbestos fiber must be taken as the lower limit. This is about 10^{-16} g (corresponding to dimensions 0.1 × 0.04 × 0.025 μm).

Once D_m is known and the mass of the largest bundle of fibers fixed, Equation 9 can be solved for the constant $(D_mC_4 + C)$ by setting $N = 1$ at the mass of the largest fiber in the population. (This is possible because of the scale-invariant nature of fractal distributions.) The number of fibers larger than the smallest mass m is then established by solving Equation 9, and the mass of the aggregate is found by integrating over the range of fiber mass. The proportion of each incremental magnitude of fiber mass is determined by multiplying the number of fibers with mean mass m times m and dividing by the mass of the aggregate.

Figure 3 was constructed by using two possible lower limits of mass, the magnitude of D_m from Table 2, and assuming that the largest particle in the population has a mass of 10^{-6} grams, corresponding to dimensions of 500 20×10 μm, which are similar to that of many finely ground industrial mineral products. Figure 3 shows the distribution of mass in monomineralic asbestos samples over the range of mass of the individual fibers.

An asbestos fiber $5 \times 1 \times 0.5$ μm is easily visible by optical microscopy. A fiber of this size has a mass of approximately 10^{-12} g. An asbestos fiber bundle 10×3 1.5 μm has a mass of about 10^{-10} g. Particles of this size are visible optically at low magnifications. Table 3 summarizes the data in Figure 3 by giving the weight percent of the total asbestos with mass equal to or greater than 10^{-14} g (fibers longer than 5 μm), 10^{-12} g (fibers and fiber bundles easily visible by optical microscopy) and 10^{-10} g (fibers and fiber bundles visible by optical microscopy at low magnification).

TABLE 3. WEIGHT PERCENT ASBESTOS IN THAT PORTION OF THE POPULATION COMPOSED OF FIBERS WITH MASS EQUAL TO OR GREATER THAN 10^{-14}, 10^{-12}, AND 10^{-10} GRAMS IN A SAMPLE WHOSE FIBERS RANGE IN MASS FROM 10^{-6} TO 10^{-16} GRAMS

Mass	Short-Fiber Chrysotile	Riebeckite-asbestos	Grunerite-asbestos	Tremolite-asbestos
10^{-14} grams	16 %	29 %	99 %	91 %
10^{-12} grams	2.6	8.4	97	79
10^{-10} grams	0.4	2.4	91	63

If the optical microscope is used to examine industrial mineral products as part of a quality-control program, or airborne or settled dusts as part of an occupational monitoring program, these data indicate that a very large proportion of the mass of tremolite-asbestos and grunerite-asbestos (and by analogy, long-fiber Canadian chrysotile) should be visible. Values of D_m can be determined by examining samples and measuring fibers by OM. The lower limit of fiber mass is then estimated, and the upper limit taken as the mass of the largest fiber found by scanning a sample of known weight at low magnification. The weight of the asbestos aggregate is determined by integrating over the range of mass. The proportion of asbestos in the sample is determined by dividing this by the weight of the sample examined. In the cases of the short-fiber chrysotile from California and of riebeckite-asbestos, much of the asbestos will not be visible by OM, and D_m must be established from TEM measurements. However, most noncommercial amphi-

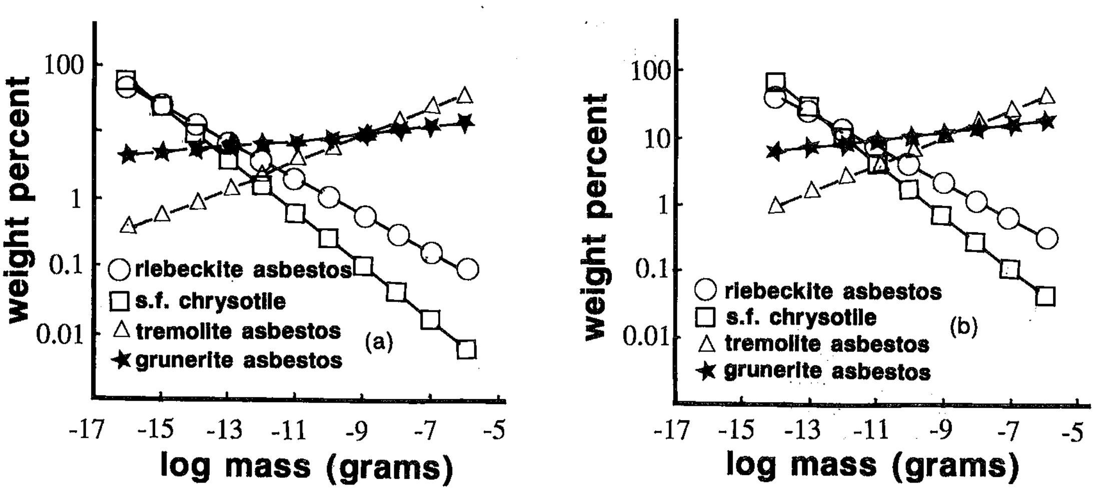

FIG. 3. Weight percent asbestos in monomineralic samples as a function of the mass of individual fibers and fiber bundles estimated from the data given in Table 2. a) Distributions assume that the smallest fiber in the population has a mass of 10^{-16} g and that the largest bundle of fibers has a mass of 10^{-6} g. b) Distribution assumes that the largest bundle of fibers has a mass of 10^{-6} g and that the smallest fiber in the population has a mass of 10^{-14} g. This distribution corresponds to the mass distribution of all fibers and bundles with lengths greater than 5 micrometers. The slopes of the lines in a) and b) are the same. Changing the assumptions about the range in a population will only change the position of the lines vertically.

bole asbestos is more likely to have dimensions similar to tremolite-asbestos and grunerite-asbestos than to riebeckite-asbestos and California chrysotile; optical microscopy will be adequate for routine screening for asbestos for most industrial mineral products.

CONCLUSIONS

The fractal dimension D calculated from log length *versus* log number of particles gives information about the friability and potential for fragmentation of asbestos. Values of D for most samples of asbestos are near or less than 1. According to Turcotte (1986), fractal dimensions of this magnitude indicate that asbestos offers a very low resistance to fragmentation. However, among asbestos samples, D is quite variable. Those with a higher value of D are likely to disaggregate to yield greater numbers of small fibers per gram than asbestos with lower D. For example, Cape and Australian riebeckite-asbestos will disaggregate to produce more abundant short particles than riebeckite-asbestos from either the Transvaal or Bolivia (Fig. 1). This behavior might be predicted since fibril width is the smallest in Cape riebeckite-asbestos and largest in Bolivian riebeckite-asbestos (Shedd 1985). Larger fractal dimensions would also be expected to occur in brittle or noncommercial asbestos, which would fracture perpendicular to elongation during grinding.

A calculated fractal dimension D_m provides information about the distribution of mass *versus* number of particles in asbestos samples. For D_m greater than 1, the proportion of the population's mass increases as length decreases. In this case, only the TEM is likely to provide adequate assessment of the mass distribution, because TEM gives a more accurate distribution than either SEM or OM of short fibers in which the mass will be concentrated. On the other hand, where D_m is less than 1, the largest fibers in a sample control the distribution of mass, and the OM or SEM is likely to provide the most important dimensional data. Anyone screening mineral samples for asbestos or administering asbestos in animal experimentation should take into consideration the magnitude of D_m in designing a method for the characterization of the samples.

The fractal dimension can be used, even qualitatively, to interpolate the abundances of fibers between the maximum length that was measured in a small random sample and the maximum length that is observed in the population as a whole. This would enable those fibers that are too uncommon to be included in a sample of a few hundred to be considered in the calculation of weight percent abundances. For the shortest fibers, the fractal dimension probably does not apply to fibers less than between 0.1 and 0.001 μm; an investigator must rely on actual measurements below this length. However, deviations from the fractal model for lengths greater than this should be considered to be due to error in sampling or measurement. Fitting the data to a fractal model enables a correction to be applied to account for such errors.

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REFERENCES

BENNETT, J. G. (1936): Broken coal. *J. Inst. Fuel* **10**, 22-39.

CAMPBELL, W.J., HUGGINS, C.W. & WYLIE, A.G. (1980): Chemical and physical characterization of amosite, chrysotile, crocidolite, and nonfibrous tremolite for oral ingestion studies by the National Institute of Environmental Health Sciences. *U.S. Bur. Mines, Rep. Invest.* **8452**.

CHISHOLM, J.E. (1973): Planar defects in fibrous amphiboles. *J. Mat. Sci.* **8**, 475-483.

DAVIS, J.M.G., ADDISON, J., MCINTOSH, C., MILLER, B. & NIVEN, K. (1990): Variations in the carcinogenicity of tremolite dust samples of differing morphology. *Occupational Safety and Health Administration.*

DORLING, M. & ZUSSMAN, J. (1987): Characteristics of asbestiform and non-asbestiform calcic amphiboles. *Lithos* **20**, 469-489.

FEDER, J. (1988): *Fractals.* Plenum Press, New York, N.Y.

HARTMANN, W.K. (1969): Terrestrial, lunar and interplanetary rock fragmentation. *Icarus* **10**, 201-213.

HUTCHISON, J.L., IRUSTETA, M.C. & WHITTAKER, E.J.W. (1975): High resolution electron microscopy and diffraction studies of fibrous amphiboles. *Acta Crystallogr.* **A31**, 794-801.

LANGE, M.A., AHRENS, T.J. & BOSLOUGH, M.B. (1984): Impact cratering and spall failure of gabbro. *Icarus* **58**, 383-395.

MANDELBROT, B. (1967): How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science* **156**, 636-638.

POOLEY, F.D. & CLARK, N.J. (1980): A comparison of fibre dimensions in chrysotile, crocidolite, and amosite particles from samples of airborne dust and from post-mortem lung tissue specimens. *Int. Agency Res. Cancer Sci., Publ.* **30**, 79-86.

SHEDD, K.B. (1985): Fiber dimensions of crocidolites from Western Australia, Bolivia, and the Cape and Transvaal Provinces of South Africa. *U.S. Bur. Mines, Rep. Invest.* **8998**.

SIEGRIST, H.G. & WYLIE, A.G. (1980): Characterizing and discriminating the shape of asbestos particles. *Envir. Res.* **23**, 348-361.

STANTON, M.F., LAYARD, M., TEGERIS, A., MILLER, E., MAY, M., MORGAN, E. & SMITH, A. (1981): Relation of particle dimension to carcinogenicity in amphibole asbestoses and other fibrous minerals. *J. Nat. Cancer Inst.* **67**, 965-975.

STEEL, E. & WYLIE, A.G. (1981): Mineralogical characteristics of asbestos. *In* Geology of Asbestos Deposits (P. Riordon, ed.). *Soc. Mining Engineers of AIME*, 93-100.

TIMBRELL, V. (1970): Characteristics of the International Union Against Cancer standard reference samples of asbestos. *In* Proc. Third Int. Conf. on Pneumoconiosis (H.A. Shapiro, ed.). Oxford University Press, Oxford, England (28-36).

TURCOTTE, D.L. (1986): Fractals and fragmentation. *J. Geophys. Res.* **91**, 1921-1926.

U.S. ENVIRONMENTAL PROTECTION AGENCY (1982): Friable asbestos-containing materials in schools; identification and notification. *U.S. Federal Register* **47**, 23360-23389.

U.S. OCCUPATIONAL SAFETY AND HEALTH ADMINISTRATION (1983): Hazardous Communication Standard. *U.S. Federal Register* **48**, 53280.

VEBLEN, D.R., BUSECK, P.R. & BURNHAM, C.W. (1977): Asbestiform chain silicates: new minerals and structural groups. *Science* **198**, 359-365.

WYLIE, A.G. (1979): Optical properties of the fibrous amphiboles. *Ann. N. Y. Acad. Sci.* **330**, 600-605.

______& SCHWEITZER, P. (1982): The effects of sample preparation and measuring techniques on the shape and shape characterization of mineral particles: the case of wollastonite. *Envir. Res.* **27**, 52-73.

______, SHEDD, K.B. & TAYLOR, M.E. (1982): Measurement of the thickness of amphibole asbestos fibers with the scanning electron microscope and transmission electron microscope. *In* Microbeam Analysis – 1982 (K. Heinrich, ed.). San Francisco Press, San Francisco, California (181-187).

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